Name: ____________________

James Ruse Agricultural High School

2023 **Year 12 TRIAL HSC EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black or blue pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I - 10 marks (pages 2 - 5)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II - 60 marks (pages 6 - 11)

- Attempt Questions 11 - 17
- Allow about 1 hour and 45 minutes for this section

Section I: Multiple choice

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 - 10.

---

1. Which of the following first order differential equation is not linear?

(A) $\dfrac{dy}{dx} = 2x^2$

(B) $\dfrac{dy}{dx} = x + y$

(C) $\dfrac{dy}{dx} = 3y^2$

(D) $\dfrac{dy}{dx} = \dfrac{3y}{x}$

2. A plane experiences two forces: $10N$ of gravity straight down, and $60N$ of thrust at an angle of $15°$ above the horizontal. What is the magnitude of the net force to the nearest Newton?

(A) $50N$

(B) $53N$

(C) $58N$

(D) $70N$

3. $\displaystyle\int \frac{1}{25+16x^2}\,dx =$

(A) $\dfrac{5}{4}\tan^{-1}\dfrac{4x}{5}+C$

(B) $\dfrac{1}{20}\tan^{-1}\dfrac{4x}{5}+C$

(C) $\tan^{-1}\dfrac{4x}{5}+C$

(D) $\dfrac{1}{5}\tan^{-1}\dfrac{4x}{5}+C$

4. A function is defined by $f(x)=\begin{cases}2, & x<3\\ x-1, & x\geq 3\end{cases}$. The value of $\displaystyle\int_1^5 f(x)\,dx$ is

(A) 2

(B) 6

(C) 8

(D) 10

5. Given $f(x)=2\sec x$ for $0\leq x<\dfrac{\pi}{2}$, then $f^{-1}(x)=$

(A) $2\cos^{-1}\dfrac{1}{x}$

(B) $2\cos^{-1}x$

(C) $\cos^{-1}\dfrac{2}{x}$

(D) $\cos^{-1}\dfrac{1}{2x}$

6. What is the multiplicity of the root $x = 1$ of the equation $P(x) = 3x^5 - 5x^4 + 5x - 3$?

(A) 1

(B) 2

(C) 3

(D) 4

7. Let $\boldsymbol{u} = 2\boldsymbol{i} + 5\boldsymbol{j}$ and $\boldsymbol{v} = 3\boldsymbol{i} + m\boldsymbol{j}$. If $|\text{proj}_{\boldsymbol{u}}\boldsymbol{v}| = |\text{proj}_{\boldsymbol{v}}\boldsymbol{u}|$, then a possible value of m is

(A) 20

(B) 4

(C) $\sqrt{20}$

(D) 2

8. Suppose a, b, c form a Pythagorean Triple where $a < b < c$. What is the maximum value the function $f(x) = \dfrac{1}{a\sin x + b\cos x + 2c}$ can attain?

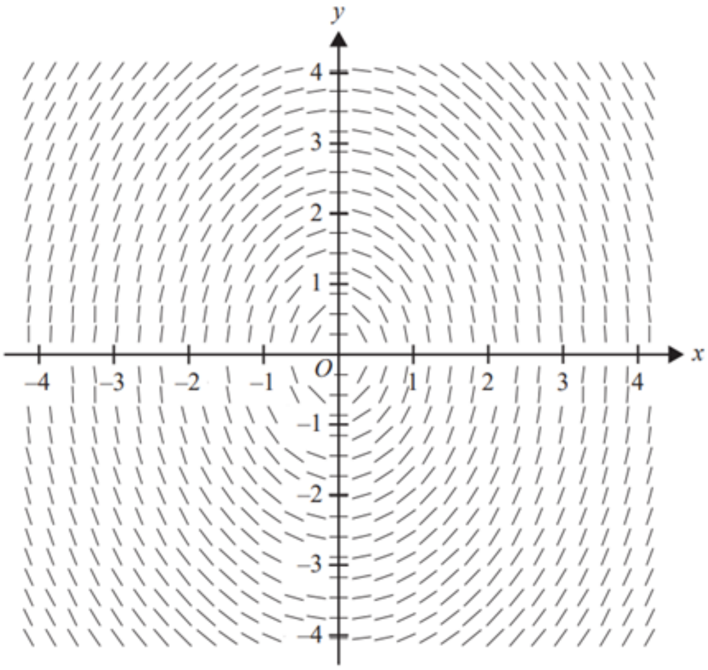
(A) $\dfrac{1}{3c}$

(B) $\dfrac{1}{c}$

(C) $\dfrac{\sqrt{2}}{(\sqrt{2}-1)c}$

(D) $\dfrac{\sqrt{2}}{(\sqrt{2}+1)c}$

9. The slope field below is best represented by the differential equation:



(A) $\dfrac{dy}{dx} = \dfrac{2x}{y}$

(B) $\dfrac{dy}{dx} = -\dfrac{x}{2y}$

(C) $\dfrac{dy}{dx} = -\dfrac{2x}{y}$

(D) $\dfrac{dy}{dx} = \dfrac{y^2}{2} + x^2$

10. Find an expression of the number of ways all the letters of the word **EPSILON** can be arranged such that the three vowels are all next to each other.

(A) 7!

(B) 5!

(C) $4! \times 5!$

(D) $3! \times 5!$

End of Section I

Section II:

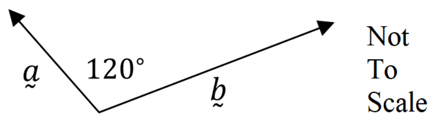
60 marks
Attempt Questions 11 to 17
Allow about 1 hour and 45 minutes for this section

Answer each question on a new page. Extra writing pages are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (9 marks) Start a new page.

(a) The diagram below shows two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, where $|\underset{\sim}{a}| = 3$ and $|\underset{\sim}{b}| = 5$. Let $\underset{\sim}{c}$ be the vector such that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$.



(i) Find $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **2**

(ii) Expand and evaluate $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b})$. **2**

(iii) Hence or otherwise, find $|\underset{\sim}{c}|$. **1**

(b) Prove by mathematical induction that, for all integers $n \geq 1$, **4**

$$\frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \frac{2}{3 \times 5} + \cdots + \frac{2}{n(n+2)} = \frac{3}{2} - \frac{2n+3}{(n+1)(n+2)}$$

End of Question 11

Question 12 (9 marks) Start a new page.

(a) Three distinct non-zero vectors are given by $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. If $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{CA}$, show that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$. **2**

(b) (i) Show that $\tan \dfrac{\pi}{12} = 2 - \sqrt{3}$. **2**

(ii) Given that $\sin(x + \alpha) = k \cos(x - \alpha)$, show that $\tan x = \dfrac{k - \tan\alpha}{1 - k\tan\alpha}$. **2**

(iii) Hence using (i) and (ii) solve, for $0 < x < 2\pi$, the equation **3**

$$(\sqrt{3} - 1)\sin\left(x + \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{3}\right)$$

.

End of Question 12

Question 13 (8 marks) Start a new page.

(a) Find k such that $\displaystyle\int_0^k \frac{1}{\sqrt{9 - x^2}}\, dx = \frac{\pi}{2}$ **2**

(b) In a large pool of people, it is known that 10% of them take less than 12 minutes to complete a particular test.

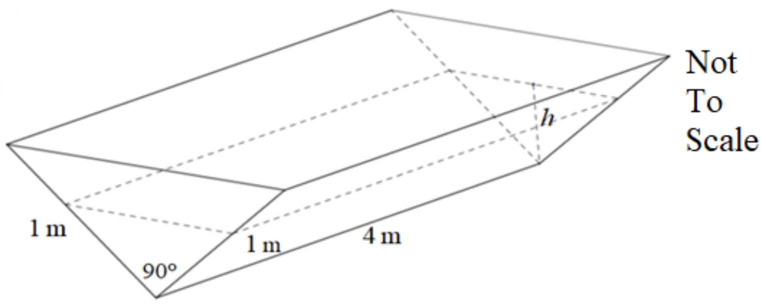
(i) 15 people are selected at random. What is the probability that fewer than 2 people will take less than 12 minutes to complete the test, to 3 decimal places. **2**

(ii) A random sample of n people is taken, the probability that fewer than 9 of these n people will take less than 12 minutes to complete the test is 0.3446, to 4 decimal places. Use normal approximation to solve for n, to the nearest integer. **4**

End of Question 13

Question 14 (8 marks) Start a new page.

(a) A four metres long water tank, open at the top, is in the shape of a triangular prism. The triangular face is a right isosceles triangle with congruent sides of one meter length, as shown below.



Initially the tank is completely full of water, but it develops a leak and loses water at a constant rate of 0.08 cubic metres per hour. Let $h =$ the depth of water, in metres, in the tank after t hours.

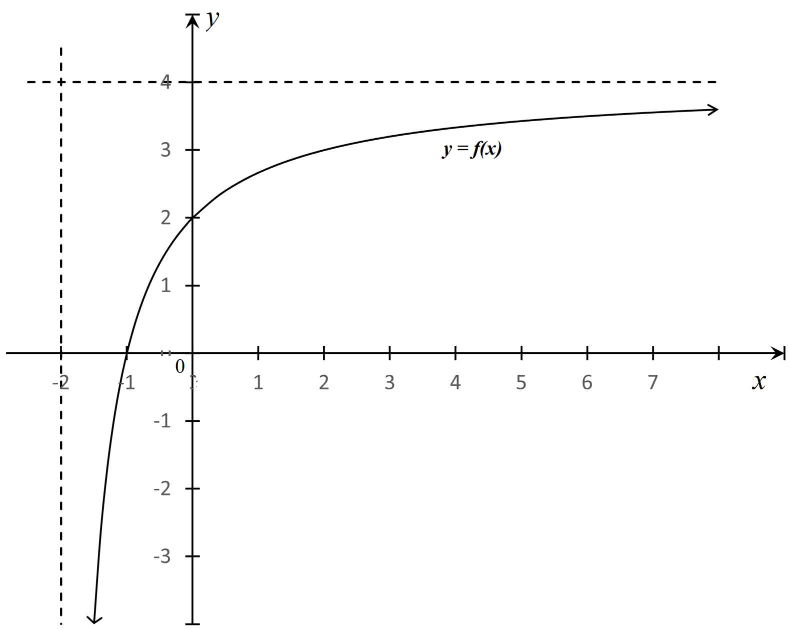
(i) Show that the volume, V, of water in the tank in cubic metres, is given by the expression $V = 4h^2$ **2**

(ii) Determine the rate of change of the depth, when the depth is 0.6 metres. **2**

(b) The diagram below shows the graph of the curve $y = f(x)$, where

$$f(x) = \frac{4x+4}{x+2}, \qquad x > -2$$

.



Question 14 continues on page 9

Question 14 (continued)

On the graph paper provided, sketch the graphs of the following curves, showing clearly the intercepts on the axes and the equations of any asymptotes.

(i) $y = |f(x)|$ **2**

(ii) $y^2 = f(x)$ **2**

End of Question 14

Question 15 (9 marks) Start a new page.

(a) Find $\int 3x^2 \left(x^3 - 1\right)^4 \, dx$ using the substitution $u = x^3 - 1$ **2**

(b) Consider the expansion of $\left(\dfrac{x^3}{2} + \dfrac{a}{x}\right)^8$. If the constant term is 5103, determine the possible value(s) of a. **3**

(c) It is known that the probability of obtaining a head when tossing a particular biased coin is 0.2. A random 100 flips of this biased coin is conducted and the number of heads are observed.

(i) Justify why the proportions of heads can be approximated using the normal distribution. **1**

(ii) Determine the mean and standard deviation of the proportion of heads. **1**

(iii) Hence or otherwise, approximate the probability that the proportion of heads obtained will be between 0.1 and 0.3 inclusive, to 4 decimal places. **2**

End of Question 15

Question 16 (9 marks) Start a new page.

(a) (i) Find $\dfrac{d}{dx}\left(\cos^{-1} x + \cos^{-1}(-x)\right)$ **2**

(ii) Hence prove that $\cos^{-1} x + \cos^{-1}(-x) = \pi$ **2**

(b) A liquid is being heated in an oven maintained at a constant temperature of $180°C$. The rate of increase of the temperature of the liquid at any time t minutes is proportional to $180 - \theta$, where $\theta°C$ is the temperature of the liquid at that time.

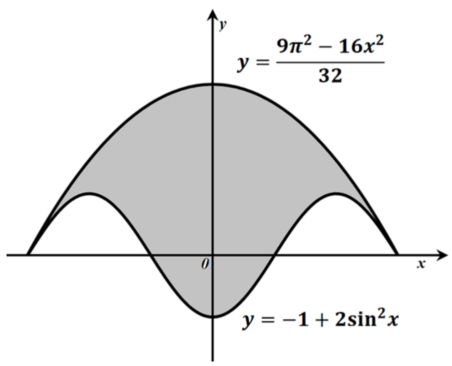
(i) Write a differential equation connecting θ and t. **1**

(ii) When the liquid was placed in the oven, its temperature was $25°C$ and 5 minutes later its temperature has risen to $75°C$. Find the temperature of the liquid after another 5 minutes. **4**

End of Question 16

Question 17 (8 marks) Start a new page.

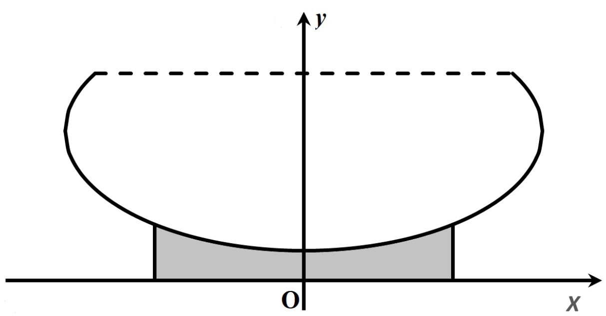
(a) The design for a company logo is depicted below. The upper border of the logo is part of the curve which has equation $y = \dfrac{9\pi^2 - 16x^2}{32}$, while the lower border is part of the curve $y = -1 + 2\sin^2 x$. The curves intersect on the $x-$axis.



(i) Solve for the $x-$ coordinates at which the two curves intersect. **1**

(ii) Determine the area of the company logo correct to 2 decimal places. **3**

(b) The diagram below shows the vertical cross-section passing through the centre of a goldfish bowl. The glass part of the bowl sits on a solid base, indicated by the shaded region on the diagram. **4**



Each horizontal cross-section is a circle. The vertical cross section passing through the centre of the bowl is symmetrical about the $y-$axis and can be described by the equation

$$\frac{x^2}{64} + \frac{(y-5)^2}{16} = 1$$

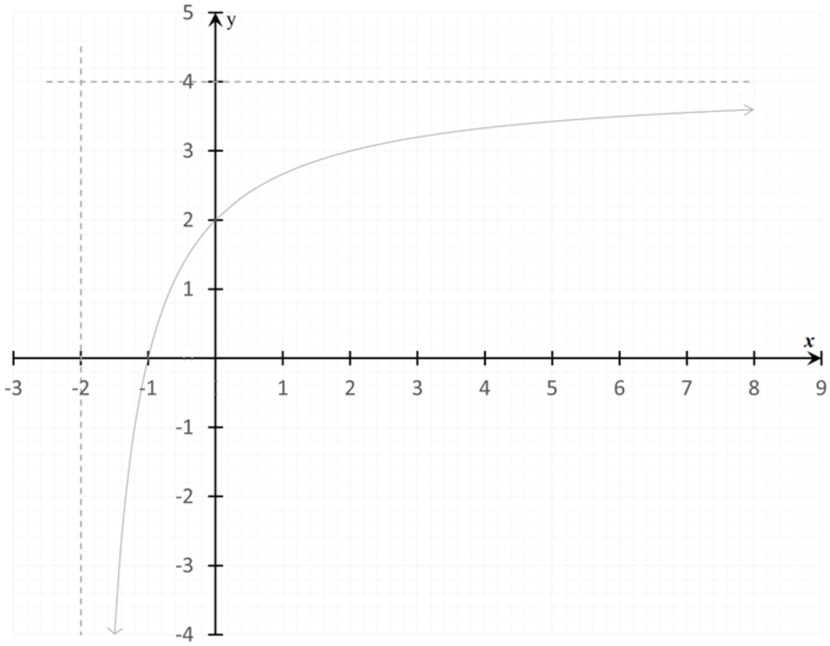
The dashed horizontal line represents the diameter of the open top of the bowl. Given that the open top has a diameter of 15cm, and that this is above the diameter of the fishbowl at its widest point, find the capacity of the glass part of the fishbowl assuming that the thickness of the glass sides of the bowl may be regarded as negligible.

End of Paper

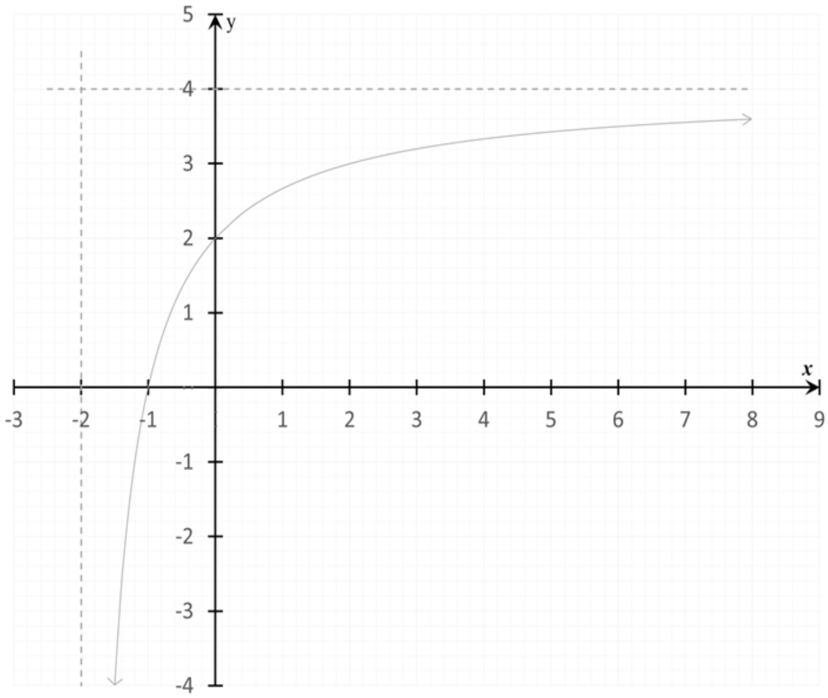
Start here for Question number: 14b

Student Number ____________________

(i) $y = |f(x)|$



(ii) $y^2 = f(x)$



2023 Year 12 Extension 1 Trials Multiple Choice

1. C
2. C
3. B
4. D
5. C
6. C
7. C
8. B
9. C
10. D

Question 11

a)

i. $\underset{\sim}{a} \cdot \underset{\sim}{b} = \left|\underset{\sim}{a}\right| \left|\underset{\sim}{b}\right| \cos 120°$

$= 3 \times 5 \times (-\cos 60°)$ (1 mark for correct subsitution into the dot product formula)

$= 15 \times -\dfrac{1}{2}$

$= -\dfrac{15}{2}$ (1 mark for final answer)

ii. $\left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} + \underset{\sim}{b}\right) = \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= \left|\underset{\sim}{a}\right|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \left|\underset{\sim}{b}\right|^2$ (1 mark for correct expansion)

$= 9 + (-15) + 25$

$= 19$ (1 mark for final answer)

iii. $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$

$\underset{\sim}{c} = -\underset{\sim}{a} - \underset{\sim}{b}$

$\left|\underset{\sim}{c}\right|^2 = \left|-\underset{\sim}{a} - \underset{\sim}{b}\right|^2$

$= \left|\underset{\sim}{a} + \underset{\sim}{b}\right|^2$

$= \left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{a} + \underset{\sim}{b}\right)$

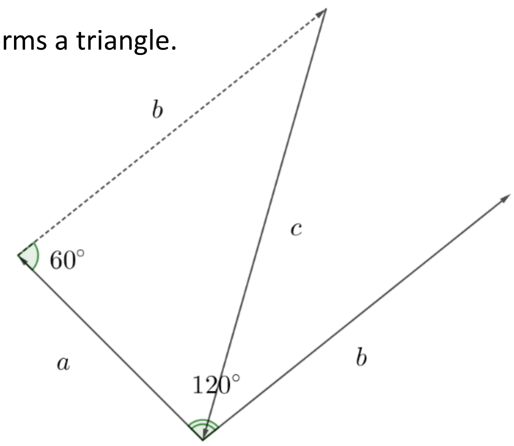
$= 19$ (from part ii)

$\left|\underset{\sim}{c}\right| = \sqrt{19}$ (1 mark for correct answer)

Alternate solution:

Construct $\underset{\sim}{b}$ from the head of $\underset{\sim}{a}$, now the vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ forms a triangle.

Using the cosine rule:



$\left|\underset{\sim}{c}\right|^2 = \left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2 - 2\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right| \cos 60°$

$= 9 + 25 - 2(3)(5)\left(\dfrac{1}{2}\right)$

$= 34 - 15$

$= 19$

$\left|\underset{\sim}{c}\right| = \sqrt{19}$ (1 mark for correct answer)

b) Test for $n = 1$:

$$\text{LHS} = \frac{2}{1 \times 3}$$
$$= \frac{2}{3}$$

$$\text{RHS} = \frac{3}{2} - \frac{2(1) + 3}{(1 + 1)(1 + 2)}$$
$$= \frac{3}{2} - \frac{5}{6}$$
$$= \frac{9 - 5}{6}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$
(1 mark for base case)

Assume the statement is true for $n = k, k \in \mathbb{Z}, k \geq 1$

$$\therefore \frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \frac{2}{3 \times 5} + \cdots + \frac{2}{k(k + 2)} = \frac{3}{2} - \frac{2k + 3}{(k + 1)(k + 2)}$$
(1 mark for making correct assumption)

Test for $n = k + 1$

$$\text{RTP:} \frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \frac{2}{3 \times 5} + \cdots + \frac{2}{k(k + 2)} + \frac{2}{(k + 1)(k + 3)} = \frac{3}{2} - \frac{2k + 5}{(k + 2)(k + 3)}$$

$$\text{LHS} = \frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \frac{2}{3 \times 5} + \cdots + \frac{2}{k(k + 2)} + \frac{2}{(k + 1)(k + 3)}$$

$$= \frac{3}{2} - \frac{2k + 3}{(k + 1)(k + 2)} + \frac{2}{(k + 1)(k + 3)} \quad \text{(By Assumption)}$$
(1 mark for using the assumption correctly)

$$= \frac{3}{2} - \frac{(2k + 3)(k + 3)}{(k + 1)(k + 2)(k + 3)} + \frac{2(k + 2)}{(k + 1)(k + 2)(k + 3)}$$
$$= \frac{3}{2} - \frac{(2k^2 + 9k + 9) - (2k + 4)}{(k + 1)(k + 2)(k + 3)}$$
$$= \frac{3}{2} - \frac{2k^2 + 7k + 5}{(k + 1)(k + 2)(k + 3)}$$
$$= \frac{3}{2} - \frac{(2k + 5)(k + 1)}{(k + 1)(k + 2)(k + 3)}$$
$$= \frac{3}{2} - \frac{2k + 5}{(k + 2)(k + 3)}$$

Therefore if the statement is true for $n = k$, then it is true for $n = k + 1$.

Since the statement is true for $n = 1$, then it is true for all integers $n, n \geq 1$. (1 mark for finishing the proof)

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
a) $\vec{OA} = \underset{\sim}{a}$ $\quad \vec{OB} = \underset{\sim}{b}$ $\quad \vec{OC} = \underset{\sim}{c}$		
$\vec{OA} \perp \vec{BC}$ $\quad \vec{OB} \perp \vec{CA}$		
$\underset{\sim}{a} \cdot (\underset{\sim}{c} - \underset{\sim}{b}) = 0$ $\quad \underset{\sim}{b} \cdot (\underset{\sim}{a} - \underset{\sim}{c}) = 0$		
$\underset{\sim}{a} \cdot \underset{\sim}{c} - \underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ $\quad \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{c} = 0$	1	
$\therefore \underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{a} \cdot \underset{\sim}{c}$ — (1) $\quad \therefore \underset{\sim}{b} \cdot \underset{\sim}{a} = \underset{\sim}{b} \cdot \underset{\sim}{c}$		
$\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{b} \cdot \underset{\sim}{c}$ — (2)		
equating (1) and (2)		
$\therefore \underset{\sim}{a} \cdot \underset{\sim}{c} = \underset{\sim}{b} \cdot \underset{\sim}{c}$		
$\underset{\sim}{b} \cdot \underset{\sim}{c} - \underset{\sim}{a} \cdot \underset{\sim}{c} = 0$		
$\underset{\sim}{c} \cdot (\underset{\sim}{b} - \underset{\sim}{a}) = 0$		
as $\underset{\sim}{b} - \underset{\sim}{a}$ is $\vec{AB}$	1	
$\therefore \vec{OC}$ is perpendicular to $\vec{AB}$		
b) i) RTP: $\tan\frac{\pi}{12} = 2 - \sqrt{3}$		
$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$		
$= \dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$	1	can also use $\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$
$= \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} \times \dfrac{1 - \sqrt{3}}{1 - \sqrt{3}}$		
$= \dfrac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$		

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
$= \dfrac{-4+2\sqrt{3}}{-2}$	1	must show working.
$= 2-\sqrt{3}$		
OR $\tan\frac{\pi}{6} = \dfrac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$		
$\frac{1}{\sqrt{3}} = \dfrac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}}$	1	
$1-\tan^2\frac{\pi}{12} = 2\sqrt{3}\tan\frac{\pi}{12}$		
$\tan^2\frac{\pi}{12} + 2\sqrt{3}\tan\frac{\pi}{12} - 1 = 0$		
$\therefore \tan\frac{\pi}{12} = \dfrac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2+4}}{2}$		
$= \dfrac{-2\sqrt{3} \pm \sqrt{16}}{2}$		
$= \dfrac{-2\sqrt{3} \pm 4}{2}$		
$= -\sqrt{3} \pm 2$		
as $\tan\frac{\pi}{12}$ is in first quadrant		
$\tan\frac{\pi}{12} \geq 0$		
$\therefore \tan\frac{\pi}{12} = 2-\sqrt{3}$	1	must justify why only positive.

MATHEMATICS Extension 1 : Question 12		
Suggested Solutions	Marks	Marker's Comments
ii) $\sin(x+\alpha) = k\cos(x-\alpha)$		
$\sin x\cos\alpha + \cos x\sin\alpha = k\cos x\cos\alpha + k\sin x\sin\alpha$	1	
divide by $\cos x\cos\alpha$		
$\tan x + \tan\alpha = k + k\tan x\tan\alpha$		
$\tan x - k\tan x\tan\alpha = k - \tan\alpha$		
$\tan x(1 - k\tan\alpha) = k - \tan\alpha$	1	
$\therefore \tan x = \dfrac{k-\tan\alpha}{1-k\tan\alpha}$		
iii) $(\sqrt{3}-1)\sin(x+\frac{\pi}{3}) = \cos(x-\frac{\pi}{3})$		
$\sin(x+\frac{\pi}{3}) = \frac{1}{\sqrt{3}-1}\cos(x-\frac{\pi}{3})$		
using part (ii)		
$\tan x = \dfrac{\frac{1}{\sqrt{3}-1} - \tan\frac{\pi}{3}}{1 - \frac{1}{\sqrt{3}-1}\tan\frac{\pi}{3}}$	1	
$= \dfrac{\frac{1}{\sqrt{3}-1} - \sqrt{3}}{1 - \frac{\sqrt{3}}{\sqrt{3}-1}}$		
$= \dfrac{1-\sqrt{3}(\sqrt{3}-1)}{\sqrt{3}-1-\sqrt{3}}$		
$= \dfrac{1-3+\sqrt{3}}{-1}$		
$= 2-\sqrt{3}$		

MATHEMATICS Extension 1 : Question 12

Suggested Solutions	Marks	Marker's Comments
$\therefore$ from part (i) $\tan x = \tan \frac{\pi}{12}$ $\therefore x = \frac{\pi}{12}, \frac{13\pi}{12}$	1 1	

Question 13

(a) $$\left[\sin^{-1}\left(\frac{x}{3}\right)\right]_0^k = \frac{\pi}{2}$$ 1 mark for correct integration.

$$\sin^{-1}\left(\frac{k}{3}\right) - \sin^{-1}(0) = \frac{\pi}{2}$$

$$\sin^{-1}\left(\frac{k}{3}\right) = \frac{\pi}{2}$$

$$\frac{k}{3} = 1$$

$$k = 3$$ 1 mark for final answer.

(b)

(i) $p = 0.1$ $\quad$ X ~ Bin(15, 0.1)

$$P(X < 2) = P(X = 0) + P(X = 1)$$

$$= {}^{15}C_0\,(0.1)^0\,(0.9)^{15} + {}^{15}C_1\,(0.1)^1\,(0.9)^{14}$$ 1 mark for correct probability expression.

$$= 0.549043\ldots$$

$$= 0.549 \;\; (3\text{dp})$$ 1 mark for correct answer.

(ii) $\mu = 0.1n$, $\quad \sigma = \sqrt{0.1 \times 0.9 \times n}$ 1 mark for correct mean and sdv.

$$P(X < 9) = P\left(Z < \frac{9 - 0.1 \times n}{\sqrt{0.09 \times n}}\right)$$

$$= 0.3466$$

$$\frac{9 - 0.1 \times n}{\sqrt{0.09 \times n}} = -0.4$$ 1 mark for correct Z-score.

$$0.01n^2 - 1.8144n + 81 = 0$$

$$n = 102.1269\ldots \text{ or } 79.313\ldots$$

$$= 102 \text{ or } 79 \text{ (nearest integer)}$$

When $n = 79$, $\mu = 7.9$, 9 is above the mean, reject.

When $n = 102$, $\mu = 10.2$, 9 is below the mean.

$\therefore \quad n = 102$ only

or $\quad 0.1n - 0.12\sqrt{n} - 9 = 0$

$\sqrt{n} = 10.1$, or $\sqrt{n} = -8.9$ (reject)

$\therefore n = 102$

1 mark for correct quadratic equation.

1 mark for correct final answer with justification.

(ii) Alternatively

$$\hat{p} = \frac{9}{n}, \quad E(\hat{p}) = 0.1, \quad \text{sd}(\hat{p}) = \frac{0.3}{\sqrt{n}}$$

1 mark for correct proportion, mean and standard deviation.

$$P(Z < \frac{\left(\frac{9}{n} - 0.1\right)\sqrt{n}}{0.3}) = 0.3466$$

$$\frac{\left(\frac{9}{n} - 0.1\right)\sqrt{n}}{0.3} = -0.4$$

1 mark for correct Z-score.

$$0.1n - 0.12\sqrt{n} - 9 = 0$$

1 mark for correct equation.

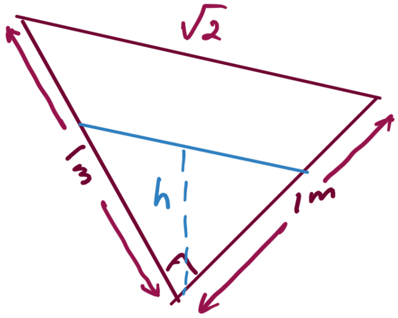
$$\sqrt{n} = 10.1, \text{ or } \sqrt{n} = -8.9 \text{ (reject)}$$

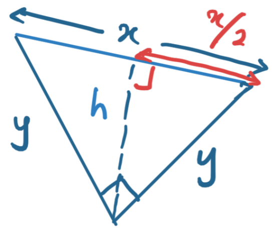
1 mark for correct final answer with justification.

$$\therefore n = 102$$

Trials 2023 **MATHEMATICS Extension 1 : Question……14..**

Suggested Solutions	Marks	Marker's Comments
a i) h - depth $\frac{dV}{dt} = 0.08\ m^3/h$		Not done well.





① - to show the base is $2h$

① - show volume

$$y^2 + y^2 = x^2$$

$$y = \frac{x}{\sqrt{2}} \quad . \ y > 0$$

$$h^2 + \left(\frac{x}{2}\right)^2 = y^2$$

$$h^2 + \frac{x^2}{4} = \left(\frac{x}{\sqrt{2}}\right)^2$$

$$h^2 = \frac{x^2}{2} - \frac{x^2}{4}$$

$$h^2 = \frac{x^2}{4}$$

$$h = \frac{x}{2} \quad h > 0$$

$$x = 2h$$

Area of Δ (water in the tank) (ie base) $= \frac{1}{2} b \times h$

$= \frac{1}{2} \times x \times h$

$= h^2$

Volume $=$ base $\times$ ht

$= 4h^2$ (shown)

MATHEMATICS Extension 1 : Question……14..

Suggested Solutions	Marks	Marker's Comments

a ii) $h = 0\cdot6 \qquad \frac{dh}{dt} = ? \qquad \frac{dV}{dt} = 0\cdot08$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$V = 4h^2$

$\frac{dV}{dh} = 8h$

$$0\cdot08 = 8h \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0\cdot08}{8\times0\cdot6}$$

$$= -0\cdot01\dot{6} \text{ or } -\frac{1}{60}$$

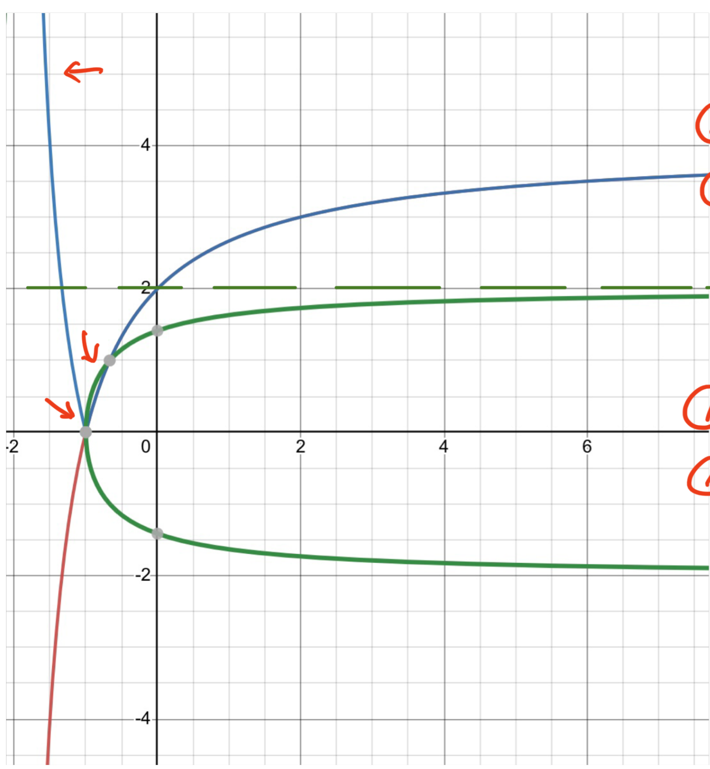
But because decreasing need to have the $-$ve sign

(1) - correct answer

(1) - negative sign or explained decreasing at $\frac{1}{60}$ m/hr

many had wrong units. No marks deducted

b) $f(x) = \dfrac{4x+4}{x+2}, \quad x > -2$



$y^2 = f(x)$ labelled

(1) - sharp point

(1) - curved inward

$y = |f(x)|$ labelled

(1) - to show the curve/bump

(1) - asymptote & shape

MATHEMATICS Extension 1 : Question 15		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $\int 3x^2(x^3-1)^4\,dx$		
$u = x^3 - 1$		
$\frac{du}{dx} = 3x^2$		
$\therefore du = 3x^2\,dx$		
$\int 3x^2(x^3-1)^4\,dx = \int u^4\,du$	1	
$= \frac{1}{5}u^5 + C$		
$= \frac{(x^3-1)^5}{5} + C$	1	must include $+ C$
b) $\left(\frac{x^3}{2} + \frac{a}{x}\right)^8$		
general term: $^8C_r \left(\frac{x^3}{2}\right)^{8-r} \left(\frac{a}{x}\right)^r$		
$= {}^8C_r\, 2^{-(8-r)} a^r x^{3(8-r)-r}$	1	
for constant term:		
$3(8-r) - r = 0$		
$24 - 3r - r = 0$		
$4r = 24$		
$r = 6$	1	
$\therefore {}^8C_6\, 2^{-(8-6)} a^6 = 5013$		
$7a^6 = 5013$		
$a^6 = 729$		
$\therefore a = \pm 3$	1	

MATHEMATICS Extension 1 : Question 15		
Suggested Solutions	**Marks**	**Marker's Comments**
c) i) $np = 100(0.2) = 20 \geq 10$ $nq = 100(0.8) = 80 \geq 10$ $\therefore$ can be approximated using the normal distribution.	1	
ii) $\mu = 0.2$ $\sigma = \sqrt{\dfrac{0.2 \times 0.8}{100}} = 0.04$	1	
iii) $P\left(\dfrac{0.1-0.2}{0.04} \leq Z \leq \dfrac{0.3-0.2}{0.04}\right)$ $= P(-2.5 \leq Z \leq 2.5)$	1	
$= 0.9938 - 0.0062$ $= 0.9876$	1	

Question 16

a, i, $\frac{d}{dx}\left[\cos^{-1}x + \cos^{-1}(-x)\right]$

$= \frac{-1}{\sqrt{1-x^2}} + \frac{-(-1)}{\sqrt{1-(-x)^2}}$

1 mark for correct derivative of cos inverse x

1 mark for correct solution

$= 0$

ii, $\frac{d}{dx}\left(\cos^{-1}x + \cos^{-1}(-x)\right) = 0$

$\therefore \cos^{-1}x + \cos^{-1}(-x) = C$

$\forall x \in (-1,1)$

sub $x = 0$,

1 mark for making the link to the previous part ("Hence")

$\cos^{-1}0 + \cos^{-1}0 = \underline{\frac{\pi}{2} + \frac{\pi}{2}}$

1 mark for correct solution, showing all the steps required

$= \pi$

$\therefore C = \pi$

$\therefore \cos^{-1}x + \cos^{-1}(-x) = \pi \quad \forall x \in (-1,1)$

Now, $\cos^{-1}(-1) + \cos^{-1}(-(-1)) = \pi + 0 = \pi$

and $\cos^{-1}(1) + \cos^{-1}(-1) = 0 + \pi = \pi$

$\therefore \cos^{-1}x + \cos^{-1}(-x) = \pi \quad \forall x \in [-1,1]$

b, i, $\frac{d\theta}{dt} = k(180-\theta)$, $k \in \mathbb{R}^+$

1 mark for correct solution

ii, $\int_0^5 k \, dt = \int_{25}^{75} \frac{1}{180-\theta} \, d\theta$

$kt \Big]_0^5 = -\ln|180-\theta| \Big]_{25}^{75}$

1 mark for separable DE

1 mark for correct integration

1 mark for finding the value of k

1 mark for the correct solution

$5k = \ln|180-25| - \ln|180-75|$

$= \ln \frac{155}{105}$

$k = \frac{1}{5} \ln \frac{31}{21}$

$-\frac{1}{5} \ln \frac{21}{31}$

$-\frac{1}{5} \ln \frac{105}{155}$

$\int_0^{10} \frac{1}{5} \ln \frac{31}{21} \, dt = \int_{25}^{x} \frac{1}{180-\theta} \, d\theta$

$\left(\frac{1}{5} \ln \frac{31}{21}\right) t \Big]_0^{10} = -\ln|180-\theta| \Big]_{25}^{x}$

$2 \ln \frac{31}{21} = \ln 155 - \ln|180-x|$

$\ln \left(\frac{31}{21}\right)^2 = \ln \frac{155}{|180-x|}$

$\frac{961}{441} = \frac{155}{|180-x|}$

$|180-x| = \frac{2205}{31}$

$\therefore x = \frac{3375}{31}$ $(x \leq 180)$

$\approx 109°$

b, ii,

$$\int k \, dt = \int \frac{1}{180-\theta} \, d\theta$$

$$kt = -\ln|180-\theta| + C$$

$(t=0, \theta=25)$

$$0 = -\ln 155 + C$$

$$\therefore C = +\ln 155$$

$$\therefore kt = -\ln|180-\theta| + \ln 155$$

$(t=5, \theta=75)$

$$5k = -\ln 105 + \ln 155$$

$$\therefore k = \frac{1}{5}\ln\frac{31}{21}$$

$$\therefore \frac{1}{5}\ln\frac{31}{21} t = \ln\frac{155}{|180-\theta|}$$

When $t=10$,

$$\frac{1}{5}\ln\frac{31}{21}(10) = \ln\frac{155}{|180-\theta|}$$

$$\left(\frac{31}{21}\right)^2 = \frac{155}{|180-\theta|}$$

$$|180-\theta| = \frac{2205}{31}$$

$$\therefore \theta = \frac{3375}{31} \quad (\theta \le 180)$$

$$\approx 109°C$$

Q17 a)

i) at $y=0$, $0 = \frac{9\pi^2 - 16x^2}{32}$

$$16x^2 = 9\pi^2$$

$$x^2 = \frac{9}{16}\pi^2$$

$$x = \pm\frac{3}{4}\pi \quad 1$$

$\therefore$ POI are $\left(\frac{3\pi}{4}, 0\right)$ and $\left(-\frac{3\pi}{4}, 0\right)$

ii)

$$A = \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{9\pi^2}{32} - \frac{16x^2}{32} - \left(-1 + 2\sin^2 x\right)\right) dx \quad 1$$

$$= \int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{9\pi^2}{32} - \frac{x^2}{2} + 1 - \left(1 - \cos 2x\right)\right) dx$$

$$= \left[\frac{9\pi^2}{32}x - \frac{x^3}{6} + \frac{1}{2}\sin 2x\right]_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} \quad 1$$

$$= \frac{9\pi^2}{32}\left(\frac{3\pi}{4}\right) - \frac{1}{6}\left(\frac{3\pi}{4}\right)^3 + \frac{1}{2}\sin 2\left(\frac{3\pi}{4}\right) - \frac{9\pi^2}{32}\left(\frac{-3\pi}{4}\right) + \frac{1}{6}\left(\frac{-3\pi}{4}\right)^3 - \frac{1}{2}\sin 2\left(\frac{-3\pi}{4}\right)$$

$= 7.720515\ldots$

$= 7.72u^2$ (2dp) $\quad 1$

Exact value : $\frac{9\pi^3}{32} - 1$

Q17b

when $x = 0$, $\frac{(y-5)^2}{16} = 1$

$$(y-5)^2 = 16$$

$$y - 5 = \pm 4$$

$$y = 9, 1$$

$y = 1$ for lower bound

when $x = 7.5$, $\frac{(7.5)^2}{64} + \frac{(y-5)^2}{16} = 1$

$$\frac{(y-5)^2}{16} = \frac{31}{256}$$

(1) for the correct upper and lower bounds

$$(y-5)^2 = \frac{31}{16}$$

$$y - 5 = \pm \frac{\sqrt{31}}{4}$$

$$y = 5 \pm \frac{\sqrt{31}}{4}$$

$\therefore$ $y = 5 + \frac{\sqrt{31}}{4}$ is upper bound

$$V = \pi \int_1^{5+\frac{\sqrt{31}}{4}} x^2 \, dy$$

$$= \pi \int_1^{5+\frac{\sqrt{31}}{4}} 64\left(1 - \frac{(y-5)^2}{16}\right) dy \quad (1)$$

$$= 64\pi \int_1^{5+\frac{\sqrt{31}}{4}} \left(1 - \frac{(y-5)^2}{16}\right) dy$$

$$= 64\pi \left[y - \frac{(y-5)^3}{48} \right]_1^{5+\frac{\sqrt{31}}{4}} \quad (1)$$

$$= 64\pi \left(5 + \frac{\sqrt{31}}{4} - \frac{\left(5+\frac{\sqrt{31}}{4} - 5\right)^3}{48} - 1 + \frac{(1-5)^3}{48} \right)$$

$= 804.734819\ldots$

$= 804.73 \text{ cm}^3$ (2 dp)

$= 804.73 \text{ mL}$ (1) for capacity

$\frac{4/5}{...}$

$96 \frac{y}{24}$
8

16×6